Kernels and Cycles’ Subdivisions in edge-colored tournaments

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Abstract

Let D be a digraph, D is said to be an m-colored digraph if the arcs of D are colored with m colors. A path P in D is called monochromatic if all of its arcs are colored alike. Let D be an m-colored digraph, a set N ∈ V(D) is said to be a kernel by monochromatic paths of D if it satisfies the following conditions: a) for every pair of different vertices u, v ∈ N there is no monochromatic directed path between them and; b) for every vertex x ∈ V(D)-N there is a vertex n ∈ N such that there is an xn-monochromatic directed path in D.

In this paper we prove that if T is an edge-colored tournament which does not contain certain cycles’s subdivision then it possesses a kernel by monochromatic paths. This results generalize a well known sufficient condition for the existence of a kernel by monochromatic paths obtained by Shen Minggang in 1988 and another one obtained by Hahn et al. in 2002. Some open problems are proposed.

Keywords: kernel, kernel by monochromatic paths, tournament.

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1 Introduction

For general concepts on digraphs we refer the reader to [1]. Let D be a digraph, V(D) (resp. A(D)) will denote the set of vertices (resp. arcs) of D. A subdigraph H of D is a spanning subdigraph if V(H)=V(D); if S ⊆ V(D) is nonempty then the subdigraph D[S] induced by S is that digraph having vertex set S and whose arc set consist of all those arcs of D joining vertices of S. An arc $z_1z_2 ∈ A(D)$ is called asymmetrical (symmetrical) if $z_2z_1 ∉ A(D)$ ($z_2z_1 ∈ A(D)$); the asymmetrical part of D (the symmetrical part of D) denoted by Asym(D) (Sym(D)) is the spanning subdigraph of D whose arcs are the asymmetrical (symmetrical) arcs of D; D is called an asymmetrical digraph if Asym(D)=D. A digraph is called semicomplete if for every two distinct vertices u and v of D, at least one of the arcs (u, v) or (v, u) is present in D. A semicomplete asymmetrical digraph is called a tournament.

An arc $z_1z_2 ∈ A(D)$ will be called an $S_1S_2$-arc whenever $z_1 ∈ S_1 ⊆ V(D)$ and $z_2 ∈ S_2 ⊆ V(D)$. By $[z_1, z_2]_T$ we denote one of the two posible arcs between $z_1$ and $z_2$. For a directed walk we will denote by $ℓ(W)$ its length. And if $z_1, z_2 ∈ V(W)$ then we denote by $(z_1, W, z_2)$ the $z_1z_2$- directed walk contained in W. We will denote by $C_n$ a directed cycle of length n. Along the paper all the paths and cycles considered are directed paths and directed cycles.

A set S ⊆ V(D) is independent if A(D[I])=∅. A kernel N of D is and independent set of vertices
such that for each $z \in V(D)-N$ there exists a $zN$-arc in $D$. A digraph $D$ is called kernel-perfect (KP-digraph) when every induced subdigraph of $D$ has a kernel and is called critical-kernel-imperfect digraph (CKI-digraph) when $D$ has no kernel but every proper induced subdigraph of $D$ has a kernel. 

$D$ is a $m$-colored digraph if the arcs of $D$ are colored with $m$ colors. Let $D$ a $m$-colored digraph. A directed path (or cycle) is called monochromatic if all of its arcs are colored alike and it is called quasimonochromatic if with at most one exception all of its arcs are colored alike. A subdigraph $H$ of $D$ is called $k$-colored if all of its arcs are colored with only $k$ colors, in particular for $k=2$ we say that $H$ is bicolor. We will say that a subdigraph $H$ of $D$ is at most $k$-colored if all of its arcs are colored with at most $k$ colors, in particular for $k=2$ we say that $H$ is at most bicolor. A subdigraph $H$ of $D$ is defined as a $(2,k-2)$-subdivision of $C_2$-bicolor if $H$ is a directed cycle of length $k$ containing a monochromatic directed path of length $k-2$ and a monochromatic directed path of length 2. A subdigraph $H$ of $D$ is called a $(1,1,k-2)$-subdivision of a 3-colored $C_3$ (or a 3-colored $T_3$) with colors 1, 2 and 3, if it is a cycle of length $k$ having a monochromatic path of length $k-2$ colored, one arc colored 2 and one arc colored 3. The closure of $D$, denoted by $C(D)$, is the digraph having $V(C(D))=V(D)$ and $A(C(D)) = \{ (u,v) \text{ colored } i \mid \text{there exists an } uv\text{-directed path colored } i \text{ contained in } D \}$. 

In [5], Sands et al. have proved that for any 2-colored digraph $D$, $C(D)$ is a KP-digraph; in particular they proved that every 2-colored tournament $T$ has a vertex $v$ such that for any $x \in V(T)-\{v\}$ there is a monochromatic path from $x$ to $v$ (i.e. $\{v\}$ is a kernel of $C(T)$). In [6] Minggang proved that for every $m$-colored tournament $T$ such that every triangle in $T$ (that is a transitive tournament of order 3 or a cycle of length 3) is quasimonochromatic, $C(T)$ have a kernel (Theorem A). In [3] Galeana-Sánchez proved that if every directed cycle of length at most 4 is a quasimonochromatic cycle then $C(T)$ is a KP-digraph (Theorem B). Finally in [4] it was defined the Hypothesis $H_s$: a) $H_3$ is the hypothesis of Theorem A of Shen Minggang, b) For $s \geq 4$ each cycle of length $s$ is quasimonochromatic and no cycle of length less than $s$ is polychromatic (it is colored with at least three colors). In that paper Hahn et. al proved the following generalization of the previously cited results of Minggang and Galeana-Sánchez: A finite $m$-colored tournament satisfying the Hypothesis $H_s$ with $s \geq 3$ admits an absorbing vertex (Theorem C). 

In this paper we obtain two conditions which imply that $C(T)$ has a kernel. The first one is a generalization of Theorem C by Hahn et al. (see [4]) and the last one is a generalization of Theorem A, a Shen Minggang’s result. We also prove that this conditions are not implied by the previously known.

Along the paper we will need the following result:

**Theorem 1.1** (Berge and Duchet [2]) A semicomplete digraph is a KP-digraph if and only if every directed cycle of $D$ has at least one symmetrical arc.

## 2 A previous Lemma

**Lemma 2.1** Let $T$ be a $m$-colored tournament. If every $C_3 \subseteq T$ is a quasimonochromatic cycle and $C(T)$ is not a KP-digraph then there exists a cycle $\gamma = (z_0, z_1, z_2 = 0, 1, 2, \ldots, p = z_0) \subseteq C(T)$ such that the following properties hold: a) $\ell(\gamma) \geq 4$, b) $\gamma \subseteq T$, c) $(z_0, z_1) \in A(T)$ with color $a$, $(z_1, z_2) \in A(T)$ with color $b$ and there exist a $z_2z_0$-path $\alpha = (z_2 = 0, 1, 2, \ldots, p = z_0)$ $(p \geq 2)$ with color $c$, $a \neq b$, $b \neq c$, $a \neq c$, let $a = \text{red}$, $b = \text{blue}$, $c = \text{black}$, d) There is no $z_1z_0$-monochromatic path in $T$ and there is no $z_2z_1$-monochromatic path in $T$, e) $(z_2, z_0) \notin A(T)$ (so $(z_0, z_2) \in A(T)$), f) Every arc
between \( z_1 \) and an internal vertex in \( \alpha \) is not black.

**Proof:** \( C(T) \) is not a KP-digraph so it follows by Theorem 1.1 that there is a cycle \( \Gamma \subseteq \text{Asym}(C(T)) \). Let \( \Gamma=\langle z_0, z_1, z_2, \ldots, z_{n-1}, z_n=z_0 \rangle \subseteq \text{Asym}(C(T)) \) be a cycle of minimal length contained in \( \text{Asym}(C(T)) \).

1. \( \ell(\Gamma)=n \geq 3 \): Because \( \Gamma \subseteq \text{Asym}(C(T)) \).
2. \( \Gamma \subseteq T \): \( \Gamma \subseteq \text{Asym}(C(T)) \) and \( V(T)=V(C(T)) \).
3. \( (z_0, z_1)\in F(T) \) has color \( a \), \( (z_1, z_2)\in A(T) \) has color \( b \), \( a\neq b \): Since \( \Gamma \) is not monochromatic (by the contrary: \( (z_0, \Gamma, z_{n-1}) \subseteq \text{Asym}(C(T)) \) is a monochromatic path. Thus \( (z_0, z_{n-1}) \in F(C(T)) \). Hence \( (z_{n-1}, z_0) \in (\text{Sym}(C(T)) \cap \Gamma) \), a contradiction), there exist two consecutive arcs in \( \Gamma \) with different color assigned. Say \( (z_0, z_1)\in A(\Gamma) \) is red and \( (z_1, z_2)\in A(\Gamma) \) is blue.
4. For any \( z_i, z_j \in V(\Gamma) \) such that \( j\notin \{i-1, i+1\} \) it holds that \( \{(z_i, z_j), (z_j, z_i)\} \subseteq A(C(T)) \):

   Let \( z_i, z_j \in V(\Gamma) \) be such that \( j\notin \{i-1, i+1\} \). Since \( T \) is a tournament, \( (z_i, z_j)\in A(T) \) or \( (z_j, z_i)\in A(T) \), without loss of generality let \( (z_i, z_j)\in A(T) \).

   Then \( \Gamma'=\langle z_i, z_j, z_{j+1}, z_{j+2}, \ldots, z_{i-1}, z_i \rangle \subseteq T \) is a cycle with \( \ell(\Gamma')<\ell(\Gamma) \). Hence \( \Gamma'\nsubseteq \text{Asym}(C(T)) \).

   And then \( (z_i, z_j)\in A(\text{Sym}(C(T))) \).
5. \( (z_2, z_0)\notin A(T) \): If \( (z_2, z_0)\in A(T) \) then \( C_3=\langle z_0, z_1, z_2, z_0 \rangle \subseteq T \) is a cycle. It follows by the hypothesis of the theorem that \( (z_2, z_0)\in A(T) \) is red or blue. If \( (z_2, z_0)\in A(T) \) is red then \( (z_2, z_0, z_1) \subseteq T \) is a \( z_2z_1 \)-monochromatic path and \( (z_1, z_2)\in F(\text{Sym}(C(T)) \cap \Gamma) \), a contradiction. If \( (z_2, z_0)\in A(T) \) is blue, then \( (z_1, z_2, z_0) \subseteq T \) is a \( z_1z_0 \)-monochromatic path and \( (z_0, z_1)\in F(\text{Sym}(C(T)) \cap \Gamma) \), a contradiction again.

   Now, by (4 and 5) there exist a \( z_2z_1 \)-monochromatic path in \( T \) of length at least 2. Let \( \alpha=\langle z_2=0, 1, 2, \ldots, p=0 \rangle \subseteq T \) be a \( z_2z_1 \)-monochromatic path of minimum length \( (p \geq 2) \).
6. \( \alpha \) is not red nor blue: If \( \alpha \) is red then \( \alpha \cup (z_0, z_1) \) is a \( z_2z_1 \)-monochromatic path in \( T \) and \( (z_2, z_1)\in F(\text{Sym}(C(T)) \cap \Gamma) \), a contradiction. If \( \alpha \) is blue then \( (z_1, z_2)\cup \alpha \subseteq T \) is a \( z_1z_0 \)-monochromatic path in \( T \) and \( (z_1, z_0)\in F(\text{Sym}(C(T)) \cap \Gamma) \), a contradiction again. Let \( \alpha \) be black.
7. \( z_1 \notin V(\alpha) \): Otherwise \( z_1 \in V(\alpha) \) and then \( (z_2, \alpha, z_1) \) is a \( z_2z_1 \)-monochromatic path in \( T \) so \( (z_1, z_2)\in \text{Sym}(C(T)) \), a contradiction.

   Let \( \gamma=\langle z_0, z_1, z_2 \rangle \cup \alpha \). Clearly \( \gamma \) satisfies the first four properties of our Lemma 2.1. Let's conclude with the following points.
8. There is no \( z_1z_0 \)-monochromatic path in \( T \) and there is no \( z_2z_1 \)-monochromatic path in \( T \): \( \{(z_0, z_1), (z_1, z_2)\} \subseteq \text{Asym}(C(T)) \).
9. Every arc between \( z_1 \) and an internal vertex in \( \alpha \) is not black: If there exists \( i, 1 \leq i \leq p-1 \) such that \( (i, z_1)\in A(T) \) (resp. \( (z_1, i)\in A(T) \)) is black then \( (z_2=0, \alpha, i)\cup (i, z_1) \subseteq T \) (resp. \( (z_1, i)\cup (i, \alpha, z_0) \)) is a \( z_2z_1 \)-monochromatic path in \( T \) (resp. is a \( z_1z_0 \)-monochromatic path), a contradiction.

\[ \square \]
3 The main results

3.1 Condition I

Definition 3.1 Hypothesis PI_k:

a) PI_3 is the hypothesis of theorem A of Shen Minggang (every triangle in T is quasimonochromatic).

b) For some fixed integer k≥4, every C_k⊂T is at most bicolor and is not a (2,k-2)-subdivision of C_2-bicolor; and every C_t⊂T (t<k) is at most bicolor (so it is not polychromatic).

Theorem 3.1 If T is a m-colored tournament which satisfies the hypothesis PI_k with k≥3, then C(T) is a KP-digraph.

Proof: If k=3 then the result holds by Theorem A of Shen Minggang (see [6]). Assume then that k≥4. Suppose that C(T) is not a KP-digraph, then by Lemma 2.1 there exists a cycle γ = (z_0, z_1, z_2=0, 1, 2, ..., p=z_0) satisfying properties (a) to (e).

1. p>k-2: From property (c) of Lemma 2.1 we have that γ is 3-colored. Thus it follows from the hypothesis of the Theorem 3.1 that p>k-2.

2. For each i with p-(k-2)≥i≥0 we have: If (z_1, i) ∈ A(T) then for every j with p>i+j(k-2)≥i, it holds that (z_1, i+j(k-2)) ∈ A(T).

Let i with p-(k-2)≥i≥0 and assume that (z_1, i) ∈ A(T). If there is a j, p>i+j(k-2)≥i, such that (z_1, i+j(k-2)) ∉ A(T) then let j_0=\min\{j \mid p>i+j(k-2)≥i\} and (z_1, i+j(k-2)) ∉ A(T). Since T is a tournament we have (i+j_0(k-2), z_1) ∈ A(T) and it follows from the election of j_0 that (z_1, i+j_0(k-2)-(k-2)) ∈ A(T) as (z_1, i) ∈ A(T) then C_k=(z_1, i+j_0(k-2)-(k-2))∪(i+j_0(k-2), z_1)⊆T is a cycle and it is at most bicolor by hypothesis and so (z_1, i+j_0(k-2)-(k-2)) and (i+j_0(k-2), z_1) have the same color and they are not black (by (f) in Lemma 2.1), hence C_k is a (2, k-2)-subdivision of C_2-bicolor, a contradiction. We conclude that (z_1, i+j(k-2)) ∈ A(T) for each j with p>i+j(k-2)≥i.

3. For each i, p≥i>k-2 we have: If (i, z_1) ∈ A(T) then for every j, p-(k-2)≥i-j(k-2)>0, it holds that (i-j(k-2), z_1) ∈ A(T). Let i be with p≥i>k-2 and (i, z_1) ∈ A(T). If there exists j, p-(k-2)≥i-j(k-2)>0, such that (i-j(k-2), z_1) ∉ A(T) then let j_0=\min\{j \mid p-(k-2)≥i-j(k-2)>0\} and (i-j(k-2), z_1) ∉ A(T). As before we have that (z_1, i-j_0(k-2)) ∈ A(T) (T is a tournament) then (z_1, i-j_0(k-2)+(k-2)=i-(j_0-1)(k-2)) ∈ A(T) (as a consequence of (2) since p-(k-2)≥i-j_0(k-2)≥0), contradicting the election of j_0.

Now we conclude the proof by analyzing the two following cases:

Case A. p=m(k-2), with m∈N and m≥2 (recall that p>k-2).

(z_1, z_2=0) ∈ A(T) so it follows from (2) that (z_1, p-(k-2)) ∈ A(T), then there exists C_k=(z_1, p-(k-2))∪(p-(k-2), α, p=z_0)∪(z_0, z_1)⊆T and since it is at most bicolor by hypothesis then we have that (z_1, p-(k-2)) ∈ A(T) is red (it is not black by (f) in Lemma 2.1). So we conclude that C_k=(z_1, p-(k-2))∪(p-(k-2), α, p=z_0)∪(z_0, z_1)⊆T is a (2, k-2)-subdivision of C_2-bicolor, a contradiction.

Case B. p=m(k-2)+r, with m,r∈N and m≥1 y k-2>r>0:

4. (z_1, p-r) ∈ A(T) and it is red: (z_1, z_2=0) ∈ A(T) so (z_1, m(k-2)=p-r) ∈ A(T) (by 2). Then C_t = (z_1, p-r)∪(p-r, α, p=z_0)∪(z_0, z_1)⊆T is a cycle of length t=r+2 with t<k. It is at most
2-colored by hypothesis, hence $(z_1, p-r)\in A(T)$ is black or it is red and we conclude it has color red because of (f) in Lemma 2.1.

5. $(r, z_1)\in A(T)$ and it is blue: $(z_0, z_1)\in A(T)$ so by (3) we have that $(p-m(k-2))=r, z_1)\in A(T)$. Then $C_t = (z_1, z_2=0)\cup (0, \alpha, p-m(k-2)=r) \cup (r, z_1)\subseteq T$ is a cycle of length $t=r+2\prec k$. From the hypothesis it is at most bicolor, so $(r, z_1) \in A(T)$ is black or blue. By (f) in Lemma 2.1 we conclude that $(r, z_1) \in A(T)$ is blue.

6. $(z_0, r)\in A(T)$: If $(r, z_0)\in A(T)$ then there exists $C_s=(r, z_0, z_1, z_2=0)\cup (z_2=0, \alpha, r) \subseteq T$ a cycle of length $s=r+3\prec k$ and it is 3-colored $(k\geq 4 \text{ and } k-3\geq r \geq 0)$, a contradiction. Then we have that there exists a 3-colored cycle $C_0=(z_0=p, r, z_1, p-r)\cup (p-r, \alpha, z_0)\subseteq T$ $(q\leq k$ as $r\leq k-3)$, a contradiction again.

Remark 3.1 The theorem 3.1 is a generalization of the Theorem B of Hahn et. al.

Remark 3.2 If we ask only that every $C_k\subseteq T$ is at most bicolor and is not a $(2,k-2)$-subdivision of $C_2$-bicolor, then the result does not hold, as shows Figure 1 (with $k=4$).

3.2 Condition II

Definition 3.2 Let $T$ be a $m$-colored tournament. We say that $T$ satisfies the property $PII_k$ for some fixed integer $k\geq 3$ if: a) there is no $(1,1,k-2)$-subdivision of a 3-colored $C_3$ in $T$, b) there is no $(1,1,k-2)$-subdivision of a 3-colored $T_3$ in $T$, and c) there is no $(1,1,t-2)$-subdivision of a 3-colored $C_3$ in $T$, with $t=k$ and $t\geq 3$.

Theorem 3.2 Let $T$ be an $m$-colored tournament. If $T$ satisfies the property $PII_k$ then $C(T)$ is a KP-digraph.

Proof: We proceed by contradiction. Suppose that $C(T)$ is not a KP-digraph, then by Lemma 2.1 there exists a cycle $\gamma = (z_0, z_1, z_2=0, 1, 2, ..., p=z_0)$ satisfying properties (a) to (f).

1. $p>k-2$: If $p=k-2$ then $\gamma \subseteq T$ is a $(1,1,k-2)$-subdivision of a 3-colored $C_3$ in $T$, a contradiction. If $p<k-2$ then $\gamma \subseteq T$ is a $(1,1,t)$-subdivision of a 3-colored $C_3$ in $T$ $(p+2\prec k)$, a contradiction again.

2. For each $i$ with $p-(k-2)>i\geq 0$ we have: If $[z_1, i]_T$ has color $a$ $(a\neq \text{black}$, by (f) in Lemma 2.1) then for each $j$ with $p>i+j(k-2)\geq i$, we have that $[z_1, i+j(k-2)]_T$ has color $a$: we proceed by induction on $j$. Let $i, 0\leq i<p-(k-2)$, be fixed. For $j=1$ assume by contradiction that $[z_1, i]_T$ has color $a$ and that $[z_1, i+(k-2)]_T$ has color $b$, with $a\neq b$. If $(z_1, i)\in A(T)$ and $(z_1, i+(k-2))\in A(T)$ then $T_k=(z_1, i)\cup (i, \alpha, i+(k-2))\cup (z_1, i+(k-2))\subseteq T$ is a $(1,1,k-2)$-subdivision of a 3-colored $T_3$ in $T$, a contradiction. If $(z_1, i)\in A(T)$ and $(i+(k-2), z_1)\in A(T)$ then $C_k=(z_1, i)\cup (i, \alpha, i+(k-2))\cup (i+(k-2), z_1)\subseteq T$ is a $(1,1,k-2)$-subdivision of a 3-colored $C_3$ in $T$, a contradiction. If $(i, z_1)\in A(T)$ and $(i+(k-2), z_1)\in A(T)$ then $S_k=(i, z_1, i+(k-2))\cup (i, \alpha, \alpha+(k-2))\subseteq T$ is a $(1,1,k-2)$-subdivision of a 3-colored $T_3$ in $T$, a contradiction. If $(i, z_1)\in A(T)$ and $(i+(k-2), z_1)\in A(T)$ then $T_k=(i, \alpha, i+(k-2))\cup (i+(k-2), z_1)\cup (i, z_1)\subseteq T$ is a $(1,1,k-2)$-subdivision of a 3-colored $T_3$, a contradiction again. Suppose that the statement is true for $j=n$ $(p-(k-2)>i+n(k-2)\geq i)$. For $j=n+1$ $(p+i+(n+1)(k-2)\geq i+(k-2))$ we proceed as in the first case.
Remark 3.3 The theorem 3.2 is a generalization of the Theorem A of S. Minggang.

Remark 3.4 If we omit the last hypothesis (c) in the Theorem 3.2 then the result will be false. To prove it consider the digraph in Figure 1 (with k=4).

Remark 3.5 It can be proved that the conditions of Theorems B, resp. C do not imply the conditions of the Theorem 3.1 neither the conditions of Theorem 3.2 (see Figures 2-5).
Figure 2 shows a digraph $D$ which satisfies the conditions of Theorem A but it does not satisfy the conditions of Theorem 3.1 with $k = 6$: $D$ is bicolor but it contains a $(2, k - 2)$-subdivision of a $C_2$-bicolor, namely $(z_0, z_1, z_2 = 0, 1, 2, 3, 4 = z_0)$. The same digraph satisfies the conditions of Theorem B (every triangle in $D$ is at most bicolor) but it does not satisfy the conditions of Theorem 3.1 (with $k = 6$).

Figure 3 shows a digraph $D$ which satisfies the conditions of Theorem C but it does not satisfy the conditions of Theorem 3.1 with $k = 3$: every $C_3 \subseteq D$ and every $C_4 \subseteq D$ is quasimonochromatic but $(z_0, z_2, z_3, z_0)$ is a $(2, k - 2)$-subdivision of a $C_2$-bicolor. Figure 4 shows a digraph $D$ which satisfies the conditions of Theorem B but it does not satisfy the conditions of Theorem 3.2 with $k = 6$: every triangle in $D$ is at most bicolor (if there exists a 3-colored triangle $T$ in $D$ then $\{f, (z_1, z_2)\} \subseteq A(T)$ with $f \in \{(z_0, z_1), (z_0, z_2 = a)\}$ but $\delta^+(z_2) = 0$ and $d$ is the only vertex adjacent to $z_0 = e$) but $D$ contains a $(1, 1, k - 2)$-subdivision of a 3-colored $C_3$, namely $(z_0, z_1, z_2 = a, b, c, d, e = z_0)$. Figure 5 shows a digraph $D$ which satisfies the conditions of Theorem C but it does not satisfy the conditions of Theorem 3.2 with $k = 5$: every $C_3 \subseteq D$ and every $C_4 \subseteq D$ is quasimonochromatic (if there exists a no quasimonochromatic $C_t \subseteq D$ (if $t \in \{3, 4\}$) then $\{(z_0, z_1), (z_2, z_1)\} \subseteq A(C_t)$ and this can not be possible) but $(z_2, z_3, z_4, z_0, z_1) \cup (z_2, z_1)$ is a $(1, 1, k - 2)$-subdivision of a 3-colored $T_k$.

4 Open problems

1. Let $T$ be a $m$-colored tournament. If there is some fixed integer $k \geq 4$ such that every $C_s \subseteq T$ ($s \leq k$) is at most bicolor then $C(T)$ is a KP-digraph.

2. Let $T$ be a $m$-colored tournament. If there is some fixed integer $k \geq 3$ such that there is no $(1, 1, k-2)$-subdivision of a 3-colored $C_3$ in $T$ and there is no $(1, 1, t-2)$-subdivision of a 3-colored $C_3$ in $T$ ($3 \leq t < k$) then $C(T)$ is a KP-digraph.

3. Let $T$ be a $m$-colored tournament. If there is some fixed integer $k \geq 3$ such that there is no $(1, 1, k-2)$-subdivision of a 3-colored $T_3$ in $T$ and there is no $(1, 1, t-2)$-subdivision of a 3-colored $C_3$ in $T$ ($3 \leq t < k$) then $C(T)$ is a KP-digraph.
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